Optimizing a 4-bar Lift

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What is a 4-bar?

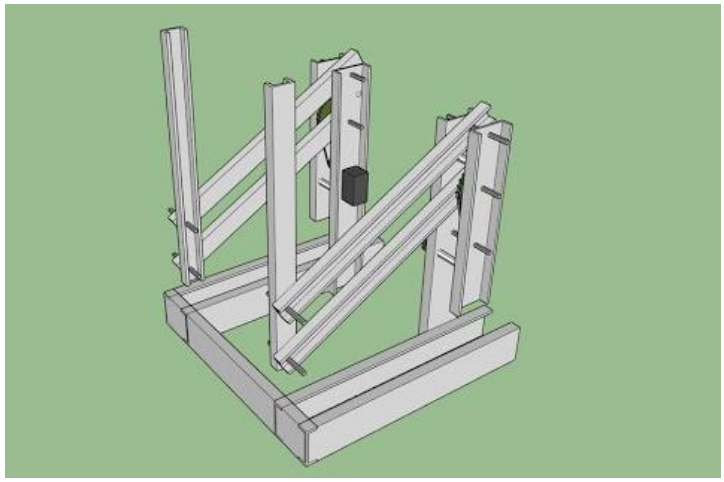


Figure 1

Throughout my time in competitive robotics, I have been introduced to many types of lifts and linkages. One of the most common lifts and linkages we see is a parallel 4-bar. This is a lift with 2 sets of parallel arms – the name is given due to the resemblance of the 4-sides of a parallelogram. On the right, you can see the prototype of a lift being made with a single 4-bar on each side. By nature a 4-bar is one of the quickest, easiest, and lightest lifts available. Due to the nature of it's design, it is incredibly simple to implement regularly, but is incredibly difficult to optimize. This is not only because robots need to start within an 18x18x18" starting block, but also because arms are limited to a maximum length of around 18". To maximize the height gained from a lift, mounting positions need to be carefully considered. In this investigation, I'm going to attempt to optimize the mounting distance of the arms, that is, how far apart they are vertically, assuming the lift is similar to the one shown in Fig. 1 above. First, however, I should explain my system and naming conventions.

Defining The System

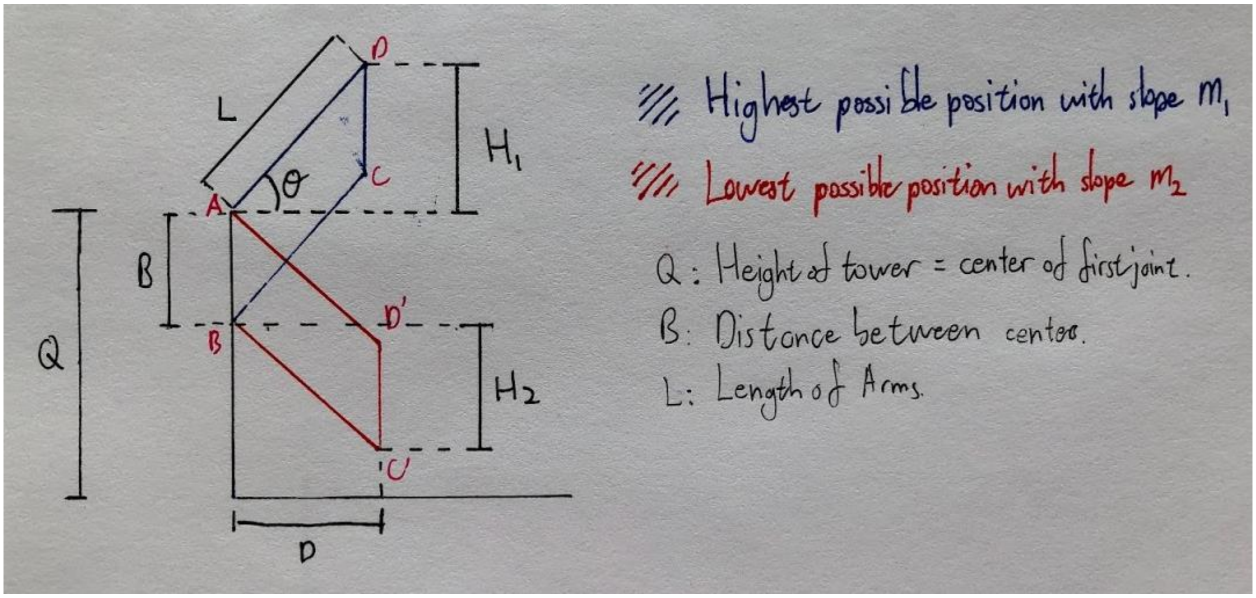


Figure 2

For my investigation, I will be using the system seen in Fig. 1, where Q is the total height from the ground up to the center of the upper axle. This is usually equal to the height of the tower, however, other circumstances may prevent this. For the sake of my investigation, I will assume that the axles are points and that the lift exists solely in a two-dimensional plane – this is because the third dimension and size of the axles is irrelevant to our investigation. B is the vertical distance between the two axles, and L is the length of the arms (the two arms have to be the same length). C' and C are the "same" points, there is the distinction of a prime so that the difference between the upper half of motion and lower half of motion can be discerned. Likewise, D' and D are the exact same point – D' denoting the lower position of the point.

Now, having described my naming conventions, I can explain why a lift needs optimizing. In an ideal world, the point A and B would be as close together as they could, resulting in a B value almost at 0. However, in real life, bars not only have a length, but also a width of around 1". In reality, holes would need to be tapped into the bar, however, I have chosen to write produce and develop this equation for ONLY the arm lengths from center to center of the axle holes as I feel this will be more useful in the future – it is quite simple to calculate the different axle sizes and spacing, and the arm lengths for this equation can easily be adjusted for the different spacing that axles would need. Creating a formula for a single specific spacing makes no sense.

Now – if we were to take the bars themselves as the lines AD/BC/AD'/BC', then each arm would need 0.5" of buffer space around it to prevent collisions with the other arm (arms themselves are 1" in this case). This means that the ideal scenario of having a near-0 B value is not physically possible. However, if you lower point B, the arm BC' start to become limited by the ground, as is illustrated in Fig. 3. When I make the calculations, we will have to compensate for the actual width of the arms colliding with the 'ground' before the actual center, but for demonstrative purposes Fig. 3 is accurate.

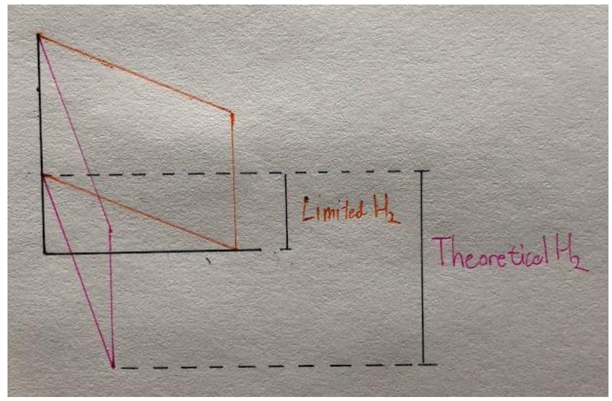


Figure 3

So, what exactly do we need to measure? Well, H_T is a measurement of total movement – where this movements starts is irrelevant, as long as it is at or above ground level. This will be explained later, but H_T can be given as the sum of H_1 and H_2, which can both be independently calculated.

Proof for Splitting H_1 and H_2

H_1 is the height travelled by the point D from when AD is first parallel to the ground to when it reaches the maximum height possible. H_2 is the distance travelled from the ground to parallel. Before I start calculating for H_1 and H_2, I think it's important to establish why adding H1 and H_2 results total height, even if H_2 doesn't necessarily touch the ground.

Proving $H_T = H_1 + H_2$

The nature of a 4-bar is that the two bars are parallel, and our total height is going to be measured from any point on CD – for the sake of argument, we could take point C and point D, and the midpoint. C and D will both travel across the same arc – the angle between those arms and the horizon will always be the exact same. This means that the arc made by C and D both have the same angle, radius, and thus same length. This length can then be split into a horizontal and vertical component, giving the total vertical displacement.

I have already established that any two parallel arms with the same length will travel the same vertical distance, and with that I can establish how any point on CD will travel the exact same distance vertically. Any point on CD is part of a line that not only passes through CD but also intersects AB. The segment of the line in between AB and CD would then not only be parallel to AD and BC, but also have the same length. As proven earlier, this means that whatever point this line intersects will travel the same distance as any other point on CD.

With that in mind, for X degrees, any point on CD should have travelled the same distance as any other. This means that the total motion is equal to the sum of all its parts. Hence taking the vertical movement of any point on CD throughout those ranges and adding them up gives the total vertical movement of the lift. H_2

Proving the Starting Position of H_2 is irrelevant

In the end, I am attempting to determine the total movement that I can gain from the lift. If H_2 starts "on the ground", then there is no issue in determining that the given H_T is in fact the actual usable range we should expect from the lift. This becomes more confusing when H2 is limited by collisions between the arms. As I have previously stated the total motion upwards can be measured by any point on line CD as they will all travel the same distance upwards. Using this argument, we can extend CD until it intersects with the ground plane, and mark that point C''. C'' will travel the same distance as C, except it has started touching the ground, and is thus "total range of motion".

M_1 and M_2

The first step in identifying the total distance travelled by the towers is determining the maximum slope that each of the arms can reach or attain. Knowing the maximum slope allows us to calculate the angle ϑ or α, where ϑ is the angle between horizon and AD, and α is the angle between horizon and BC'. Knowing these angles, we can find the height travelled. With this height known, all we need to do is graph!

Determining M_1

When solving for M_1, we set the line created by BC as y=mx, and thus the second line AD to be y=mx+b. From there, we know that one point on the second line must be (0,b), and thus BC has equation 0 = mx-y. From there, we can use the equations for distance between a point and a line to give the formula 1=b/rt(m^2+1), which can be re-arranged to m=rt(b^2-1).

Determining M_2

M_2 is more difficult to determine, and involves the creation of another triangle, and the introduction of another point. As the arms themselves have a thickness, we need to understand that this will collide with the ground before the actual "center". With that in mind, we add a point "E", which is the edge of the arm, whatever material that may be, with CE being half the width of the entire arm .

BCE then, forms a right triangle with hypotenuse BE. This triangle will remain constant throughout – as the points are relative to the tube stock itself. BE can be given from L and the width of the tube stock, giving it length $BE = \sqrt{L^2 - CE^2}$.

Knowing this, we can write the equation: $(Q-B)^2+D^2=BE^2$, where D is the horizontal distance to point E, Q is the total height, and B is the separation between centers. This can be re-arranged to give $D=\sqrt{BE^2-(Q-B)^2}$. Knowing that the slope is given by rise over run, we can calculate the slope for BE as m3, which we can use to find the slope of BC m_2. Slope is rise over run, which means that $m_3=\frac{Q-B}{D}=\frac{Q-B}{\sqrt{BE^2-Q^2+2QB-B^2}}=\frac{Q-B}{\sqrt{L^2-CE^2-Q^2+2QB-B^2}}$.

I also need to be able to express m_2 in terms of m_3, which I decided to do by comparing angles. Allow the angle between horizon and BE to be ω, and the angle EBC to be γ. α then, would be $\omega-\gamma$, this gives us an equation for m_2 of:

$$m_2=\frac{m_3-\frac{CE}{L}}{1+\frac{m_3 CE}{L}}, \text{ where m3 is } \frac{Q-B}{\sqrt{L^2-CE^2-Q^2+2QB-B^2}}.$$

This can then be put into a system of equations with m_1 to find the crossover point, a B value where the two slopes are equal.

The Floating Case

Sometimes, there will be an instance where the linkage does not have enough range of motion to touch the ground. This however, does not affect us in any way – we are only using the slope m_2 as a comparison tool to m_1, not as an actual indicator for height (like m_1). In order to understand why this doesn't matter, it's important to consider how we get m_3, and how we use m_3 and m_2.

First, m_2 and m_1 are compared against each other in order to determine where the physical limit for the part is, that is, what the actual H_2 should be. If m_2 is greater than m_1, then it suggests that the limit provided the ground is lower than the limited provided by the distance between the channels themselves, and thus $H_2 = H_1$. However, if m_2 is less than m_1, it suggests that the ground is the limit, not the distance between the c-channels.

Second, when m_3 is calculated, it is done so by using the vertical distance, length of the arm, and supposed horizontal distance travelled. This means that as B approaches zero, the slope of the line will be steeper. At some point, the slope of m_2 should equal to that of m_1, which means that the arm is limited by both the ground and by the material thickness. Any steeper, and height will be limited by m_1. As B continues to shift upwards, then the height is governed by m_1. In fact – m_2 continues to steepen, which leads us to take values for H_2 based off m_1, meaning that we don't need to account for this difference.

Final Calculations for Total Height

Finally, this leads us to calculations for total height. There are two scenarios, already outlined, one where $m2>m1$, and one where $m2<m1$.

$M_2>M_1$

When $m_2>m_1$, then $H2 = Q – B$. This is because m_2 is steeper than m_1, meaning that the arms are going to be limited by their actual thickness, which means that our final height H_T can be described as two times H_1 (because they're limited in the same way on each side), and thus by the equation:

$$H_T = 2L\frac{\sqrt{B^2 - 1}}{B^2}$$

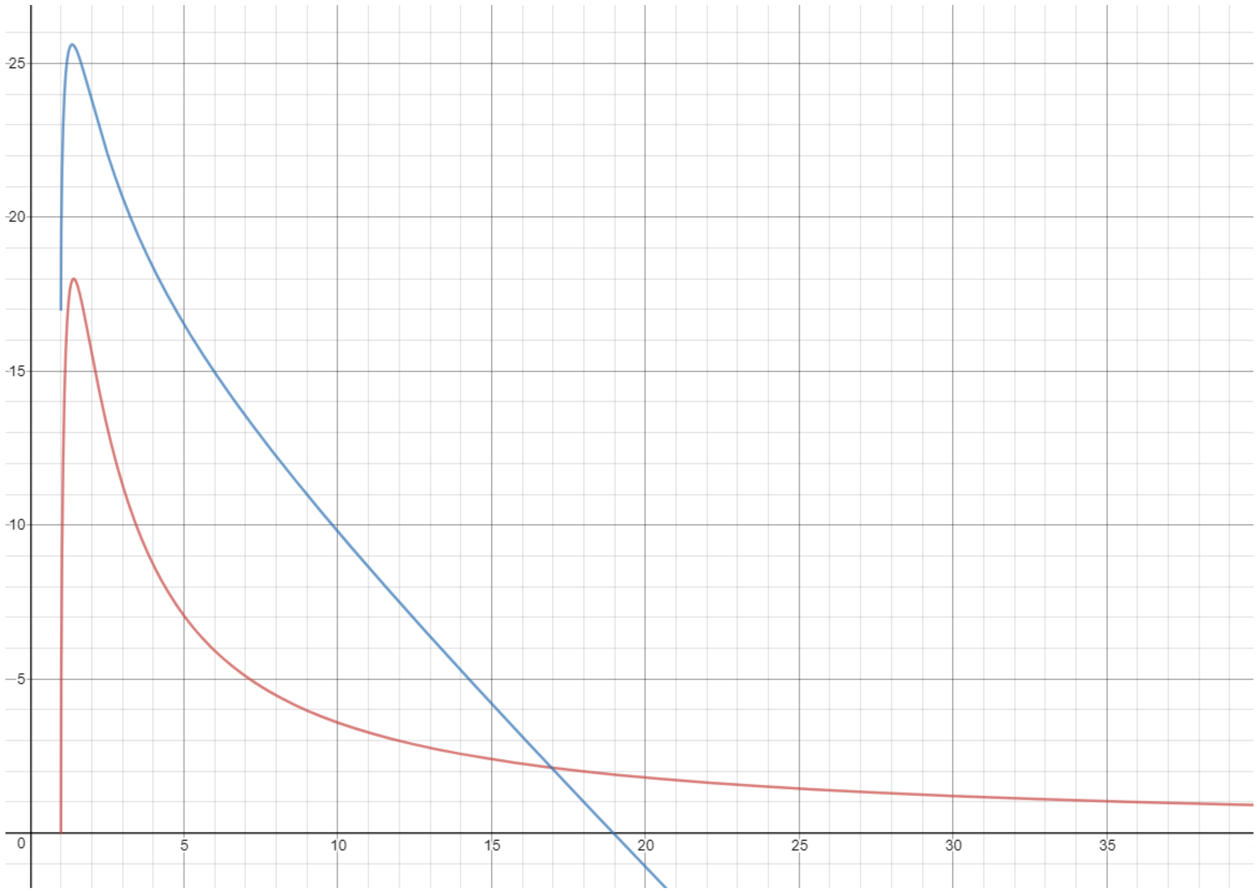
$M_2<M_1$

When $m_2< m_1$, then $H_2 = H_1$, and our final height H_T can be described in the equation below:

$$H_T = L\frac{\sqrt{B^2 - 1}}{B^2} + Q - B$$

Our Values

Finally, we can substitute our values for L, and Q into the two equations to create the following graph:



Red being the graph for $m_2 > m_1$, and blue being the graph for $m_2 < m_1$. Obviously, we need to find the point where we switch from one graph to another – calculating the value of B where $m_2 = m_1$ gives us a B value of around 17, which makes sense according to the graph. This means that we should be looking at the red line, for maximum height. In our case, we can find the max of the equation $H_T = 36\frac{\sqrt{B^2-1}}{B^2}$ by finding where the derivative crosses 0. This gives us a point of (1.37,25.6). which suggests a maximum lift height of around 25”, with a vertical offset of around 1.4”.